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COALITION STRUCTURES, DUVERGER'S LAW, AND THE 'SPLIT-MERGER STABILITY' HYPOTHESIS

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(summary)

Marek Kamiński

The first part of the paper is devoted to developing a non-spatial, game-theoretic framework capable of capturing the complexity of electoral coalition formation process. The second part focuses on the analysis of the Polish 1993 parliamentary elections in the light of the introduced framework.

The game-theoretic model developed here describes coalition formation in the context of parliamentary elections as a special type of cooperative games called coalition structure form games (CS-games). Players in CS-games can form coalitions and the partition of the set of all players defines a corresponding coalition structure. The payoffs of a coalition are defined in terms of legislative seats. Parties are assumed to form different coalition structures in order to maximize their expected numbers of seats.

The solution concept adopted here is called 'split-merger stability'. A coalition structure, or a party system, is 'split-merger stable' if, in essence, parties cannot profit from a split or a merger under the adopted voting rule and actual preferences of the electorate. The 'split-merger stability' hypothesis is formulated on this basis and states that: "An electoral system working for a long period of time under an unchanged voting rule and inside a relatively stable political environment, will develop a split-merger stable party system."

The above hypothesis can be tested for any electoral system satisfying the assumptions of a stable political environment and an unchanged voting rule. One of the possible conjectures which can be formulated on the basis of this hypothesis is that some

adjustment towards greater stability can be expected from any state of 'initial instability'. In other words, there is a tendency for parties in such a state to begin coalescing, merging, or splitting to consume all the possible gains from such activity.

The analysis of electoral results of rightist parties in 1993 Polish elections suggests that the outcome was highly 'split-merger' unstable. The analysis shows how the shift in electoral preferences and the change of the voting rule have contributed to the final instability. Moreover, the coalescing can be expected between or among parties which could profit most.

The possible operationalization of this conjecture involves a project of measuring such profits with a means of a survey data and a simulation software and monitoring the process of real coalition making at the same time. For the methodology of such research is only generally described, the empirical part of the paper includes only a documented suggestion that the result of 1993 elections was split-merger unstable and describes a possible empirical test. The whole research will be completed after the data will be collected and the final version of the software prepared.

COALITION STRUCTURES, DWERGER'S LAW, AND THE 'SPLIT-MERGER STABILITY' HYPOTHESIS[1]

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1. Introduction. The model developed here describes coalition formation in the context of parliamentary elections as a special type of cooperative games called coalition structure form games (CS-games). Players in CS-games can form coalitions and the partition of the set of all players defines a corresponding coalition structure. The payoffs of a coalition depend on the coalition structure actually formed and they should be interpreted as the actual payoffs (under the given coalition structure) rather than the minimal guaranteed payoffs (as in the case of coalition or characteristic function form games [C-games]). "Coalitions" of parties or "coalitions of coalitions" can be understood as "parties which merged". The "coalition structure" with k coalitions will be referred to as a k-party system. Mergers and splits between and among seats-maximizing parties are assumed to be the major forces influencing the formation of party systems.

The framework adopted here separates the influence of electoral support (or popular votes) (p) and voting rule (or electoral laws) (l) their effect on the motivations of parties to coalesce and eventually to form a "stable" party system. "Electoral support" here denotes all the relevant information about the distribution of declared preferences in the electorate (i.e., including possible strategic misrepresentation of preferred weak candidates etc.). Section 2 presents the game-theoretic tools used in the analysis, and Section 3 covers a general presentation of the constructed model.

In the subsequent analysis a hypothesis, called 'Split-Merger Stability Hypothesis', which is related to the stability of every party system, will be derived from the CS-game structure. The 'Split-Merger Stability' hypothesis is a refmement of the famous "Duverger's Law" and "Duverger's Hypothesis". It is conjectured that it holds when the voting rule l is constant over a longer period of time, and no serious changes of electoral preferences take place. The relations with the Duverger's Law and with some documented empirical deviations will be presented in Section 4. Section 5 turns to PR-electoral systems and describes coalition failures and their impact on fragmentation of the Polish parliament after September 1993 elections. Section 6 offers a possible operationalization of the 'Split-Merger Stability' hypothesis.

Let me describe some important underlying assumptions. Spatial positions taken by "parties and their coalitions" are implicitly assumed to be constant; parties/coalitions are seats-maximizers and have full information about the distribution of popular votes; and the number of parties is exogenously given. These are strong assumptions which undoubtedly limit the validity of the model. However, some of them are weaker than those in spatial models, e.g., the informational requirements regarding preferences of the voters and their ability to process information are less demanding.

Practical applications of the model developed below require the utilization of both detailed survey data and complicated calculations, not possible with standard mathematical statistical packages or spreadsheets. Therefore, the model is implemented here by a piece of software, taking ballot data ($\underline{p}$) as an input and allowing for some variability in the voting rules ($\underline{l}$).[2]

2. CS-games and !&stability. The CS-game concept was devised in the 1960s, but it was soon forgotten.[3] A CS-game consists of $\underline{n}$ players (it makes sense to assume that $\underline{n} \geq 3$), and payoffs. The payoffs are assigned to possible coalitions under the assumption that <u>other concrete coalitions have formed</u>. For a given coalition, payoffs in a CS-game depend on the whole coalition structure. Such a game provides us with a more subtle tool for investigating the coalition formation process than a characteristic function form, or C-game. Indeed, C-games can be thought of as special cases of CS-games. Most of the traditional notions (e.g., imputation, domination, transferable utility (TU) and nontransferable utility (NTU), and solution concepts) can be applied to CS-games in a manner similar to C-games.

Formally, the coalition <u>structure</u> for an a-person game is any partition of players into disjoint, nonempty, and exhaustive subsets $\underline{C}=\{\underline{C}_1, \ldots, \underline{C}_r\}$. For a given structure $\underline{C}$, such that $\underline{C}_i \in \underline{C}$, and under the assumption of TU, **CS-payoff** function $\underline{F}$ assigns to a coalition $\underline{C}_i$ a real number $\underline{F}(\underline{C}_i,\underline{C})$. The payoff defined by the function $\underline{F}$ should be interpreted as the <u>exact</u> payoff a coalition $\underline{C}_i$ receives given that the coalition structure $\underline{C}$ has formed,

As an example of an event modelled by a CS-game take Napoleon's infamous last battle. Combined forces of Wellington and Blücher defeated Napoleon at Waterloo on June

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[2] For a simplified questionnaire for the post-electoral survey see Appendix A. The program <u>SEATS</u> for data analysis was designed by the author and written by Jarek Gryz in C. It works for single-nontransferable vote (SNTV) systems and a number of different formulae, thresholds, etc.

[3] CS-game was introduced in Thrall (1962) under the name of "generalized characteristic function form game", and it was later changed to "partition-function form game" (Thrall and Lucas, 1963). Other papers on CS-games include Lucas (1963, 1966, 1967), Maschler (1963), Myerson (1977), and Lucas and Maceli (1977). See also the first edition of Owen's (1968) book and Shubik (1982:354-6).

18, 1815. Historians speculate that Napoleon's army was quite capable of defeating Wellington's and Bliicher's armies senaratelv, i.e., before they merged. Therefore, we can speculate that the payoffs for these two different coalition structures could be different. For example, Napoleon's payoffs could be as follows: F{{N};{N},{W,B}} = 0, F{ { N} ; {N} , (W} , {B}} = 1, where the payoff is for the coalition before the semi colon given that the coalition structure after the semi colon have formed. The information captured by the CS-game in this case would be lost in the C-game model.

After the form of the game was chosen, the next step is to **define** a notion of "stability" relevant to the **modelled** reality. The notion of an equilibrium, even older and more forgotten than CS-games, is similar to the one introduced by **Luce** (**1954).**[4] In the present formulation, it makes important use of the actual coalition structure. Informally speaking, $\underline{\Psi}$ is a rule which specifies admissible coalition changes **from a** given coalition structure $\underline{C}$. In other words, $\underline{\Psi}$ defines for each $\underline{C}$ a set $\underline{\Psi}(\underline{C})$, which is a subset of $\underline{\Pi}$, the set of all possible coalition structures. It is assumed that $\underline{C} \in \Psi(\underline{C})$. The imputation x is available under $\underline{C}$ if for all $\underline{C}_i \in \underline{C}$, $\Sigma_{k:k\in Ci} x_k = \underline{F}(\underline{C}_i, \underline{C})$). The structure $\underline{C}$ is **Ψ-stable** if (a) there is no coalition structure which could form and which makes all the new coalitions better off: there is an imputation x available under $\underline{C}$ such that for all $\underline{C}^* \in \underline{\Psi}(\underline{C})$ and all imputations y available under $\underline{C}^*$, there is at least one player j in some coalition in $\underline{C}^*$, different from all coalitions in $\underline{C}$, who gets strictly less than under $\underline{C}$: y_j C x_j, and (b) every coalition $\underline{C}_i \in \underline{C}$ must get strictly more than the sum of its players' security levels:.
$\underline{F}(\underline{C}_i, \underline{C}) \geq \Sigma_k \min_S \{\underline{F}(\{\underline{k}\}, \underline{S}) : \{\underline{k}\} \in \underline{S} \;\&\; \underline{k} \in \underline{C}_i\}$.

The existence and extent of stability is obviously dependent on the definition of $\underline{\Psi}$, which specifies the coalition structures both attainable and non-attainable from a given structure $\underline{C}$. If for any $\underline{C}$ only the same structure $\underline{C}$ is attainable, then (given that condition (b) is satisfied), $\underline{C}$ is trivially -stable. On. the other hand, if for all $\underline{C}$ every coalition in $\underline{\Pi}$ is attainable, then $\underline{\Psi}$ defines a kind of stability resembling that of the core in C-games. In this case function $\underline{\Psi}$ can be called **total**. However, there are problems with this kind of stability similar to those with the core in C-games: for constant-sum CS-games, under the assumption of essentiality, there are no totally stable coalition structures. The less

[4] See also **Luce (1955a, 1955b)**, **Luce &** Raiffa **(1957)**, Owen **(1968)**, Shubik (1982). For an **application** to congressional voting in a two-party system see **Luce** and Rogow (1956).

demanding condition adopted here allows for snlits or mergers of any different existing coalitions at a given time only: parties cannot abandon an existing coalition to merge with an external coalition. An example may be illuminating. For the coalition structure {{1,2},{3},{4}} all the coalition structures attainable from this structure are listed below.

C	Ψ(C)
{1,2},{3},{4}	{1,2},{3},{4} {1,2},{3,4} {1,2,3},{4} {1,2,4},{3} {1,2,3,4} {1},{2},{3},{4} {1},{2},{3,4}

Table 1. The value of a splir-merger Ψ-function in 4-player CS-game for C={{1,2},{3},(4)}.

Therefore, the coalition structure { { 1},{2,3},4}} is not attainable from the structure {{1,2},{3},{4}}, because player 2 at the same time has abandoned his coalition with player 1 and has created a new one with player 3.

-function was originally interpreted as "constraints in society limiting changes in coalition structures [which] are to a large degree non-rational" (Luce and Raiffa 1957:220). This is not the only possible interpretation: Ψ may also express extremes of transaction costs of multilateral bargaining, as well as legal (like anti-trust law) or information constraints on coalition formation. In our case, the split-merger rule restricts the coalition formation in a fashion similar to the formation of new parties by mergers or splits. Hence the notion of a "coalition" should be interpreted here as close in the meaning to the notion of "merged parties". -stability means optimal payoffs for all coalitions with restrictions on coalition formation process imposed by Ψ. In the case of split-merger stability this means that there are neither coalitions which could increase their total number of seats by splitting, nor by merging.

3. Elements of the model. There are n-players (parties) competing for seats in m districts. The CS-game in such an electoral system can be divided into two components: popular votes p and voting rule l.[5]

The information recorded in p summarizes the actual distribution of electoral preferences in a given electoral system under all possible coalition structures. Hence it assigns to every possible coalition structure C a complete description of ballots delivered by voters during the elections under this structure. "Actual distribution" means that p records only ballot information as it appears in elections, be it obtained from sincere or sophisticated voting. All possible values of p constitute all possible distributions of actual preferences.

The l function summarizes the formal rules of converting popular votes into parliamentary seats: electoral formulae, electoral thresholds, number of districts, numbers of seats assigned to districts, etc.

One can think about p as of a database with information collected from the voters sufficient for assigning seats under a given set of possible voting rules and under varying coalition structures. With a single nontransferable vote systems and multidistrict elections p can be conveniently interpreted as a matrix of conditional (in districts) distributions of frequencies of votes for a given coalition structure C. The element p_{ij} of the matrix corresponds to the proportion of votes obtained, by a party (coalition) i in a district j. The proportions of popular votes obtained by i nationwide will be denoted as p_{i0}. The voting rule l thus assigns to every popular votes matrix a vector denoting the distribution of legislative seats. Usually there are some constraints being imposed on l, reflecting specific "desirable" properties of a voting rule, like monotonicity, impartiality, symmetry etc.

The two components of the model, which eventually lead to a CS-payoff function, have natural interpretation. The popular votes function p carries information about electoral support in a society. The "behavior" of p, when switching from one coalition to another, summarizes the influence of electoral variables relevant to the preferences of the electorate over this particular constellation. If a fascist party were to create a coalition with a communist one, one may expect they would loose a lot of votes. On the other hand, when two parties, close in the issue space, coalesce, one may expect that their votes in every district will approximately add up independently on other coalitions or parties behavior. In this special case: $p_i+p_j=p_{\{ij\}}$, independently of the behavior of others and with the summation

[5] The general idea behind the present model was recognized in the very first paper on CS-games by Thrall (1962158); "In the political arena [subadditive payoffs of two players induced by the CS-payoff function] might represent clashing ideologies [. . .] in which an open union would rather weaken support for both [. . .]. "

operation defined over vectors of proportions in districts.

If the above equality holds true for all pairs of coalitions (including "singleton coalitions" of the form: $\{\underline{i}\}$), and popular votes of other parties remain unchanged, then we can call $\underline{p}$ additive. Function $\underline{p}$ is always additive for simpler settings of apportionment situations[6], but not necessarily for multiparty electoral races. If equality in our equation is replaced by " $\leq$ ", parties $\underline{i}$ and $\underline{j}$ can benefit from coalescing in terms of popular electoral support in districts ($\underline{p}$ is superadditive). With " $\geq$ " substituting for parity, they loose some popular votes support ($\underline{p}$ is subadditive).

The voting rule $\underline{l}$ converts the proportions of votes obtained by parties or coalitions of parties into parliamentary seats. The use of a concrete voting rule usually makes a significant difference for the CS-payoff function and the outcome of the elections. As one element of these rules, PR electoral formulae for converting popular votes into seats were studied carefully by Balinski and Young (1982). Among the formulae currently in use are those devised by Hare (also known as Hamilton or "Largest Remainder"), d'Hondt (Jefferson or "Greatest Divisors"), and Saint Laguë (Webster or Major Fractions). It is important to observe here that PR formulae have different properties of encouraging (or discouraging) coalitions. Adopting one or another formula changes the CS-payoff function of a game, holding other electoral variables constant. In the literature on electoral laws, Rae (1967, 1971) attributes more importance to those formulas than Lijphart (1990), and the question as to how much PR formulae matter remains unanswered. However, manipulating the second most important element of an electoral system, the magnitude of an electoral district, is known to lead to dramatic changes in the CS-payoff function of a game, Specifically, switching to a single-member district system leads to a CS-game which provides convincing justification for Duverger's Law (Duverger, 1951). Also other properties of a voting rule, such as thresholds, will have some influence on the CS-payoff function.

In summary, assignment. of proportions of popular votes to different coalitions under different coalition structures, which constitutes the first step in setting up the CS-game, is a matter of using the empirical properties of the electoral system. Maximization of parliamentary seats - not proportion of votes - is the parties' objective. This is done by

[6] In this context, $\underline{p}\{\underline{i}\}$ is interpreted as the population of the state $\underline{i}$ (see Balinski and Young (1978, 1982)). In fact. their model of apportionment can be regarded as a special case of the model presented here, i.e., with one large district and the assumption of additivity of the $\underline{p}$ function.

using the electoral rule as a "conversion device". This conversion constitutes the final description of the CS-game: assignments of numbers of seats to coalitions, according to coalition structures (see Figure 1).

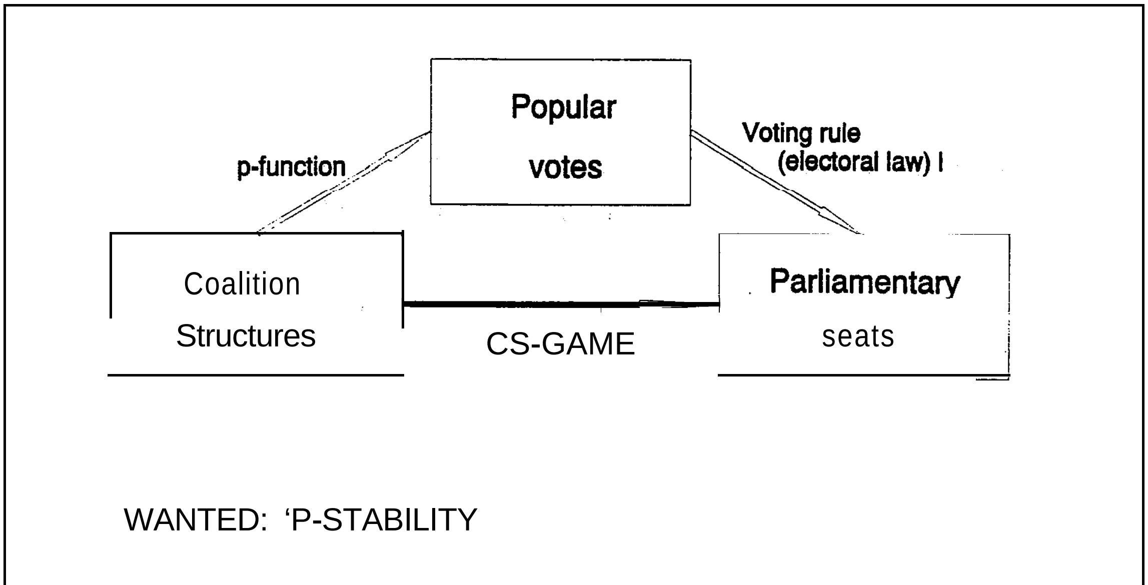

Figure 1. The graphical scheme of the model.

The most general formulation of the hypothesis regarding electoral stability in CS-games is as follows:

All electoral systems working for long periods of time under unchanged voting rules and inside relatively stable political environments, tend to produce split-merger-stable electoral systems.

Even if this version of the 'Split-Merger Stability' hypothesis uses the special formulation of the Ψ function, it should be noted that the benefits **from modeling processes** of electoral splitting and merging by the CS-games are 'independent from this particular notion of the stability. The likely better specification of the notion of stability is a subject for empirical research rather than speculation.

4. Explaining deviations from Duverger's Law. Duverger in his (1951) book noticed that "the simple-majority single-ballot system favors the two-party system. " This slightly ambiguous statement was formulated in different fashions even before Duverger's seminal piece; the ambiguity allows both for deterministic and probabilistic interpretations.

Empirical support for the deterministic version is strong, although some exceptions (like Canada or India) call for adjustments. One of the popular ways of explaining exceptions is to apply the Duverger's Law to districts only. In probabilistic formulation, Rae (1971) treated plurality voting and districts size as two "electoral variables" that contributed to the final outcome: concrete electoral system.[7]

The CS approach allows one to find a common theoretical perspective for both formulations of Duverger's Law and to extend the theory, to electoral systems under different voting rules. In the light of the 'Split-Merger Stability' hypothesis, it is also possible to find distributions of popular votes in districts @-function) and to indicate other forces responsible for possible deviations from Duverger's Law.

Below, I will analyze two examples of electoral systems working under three single-member districts, no thresholds, plurality voting rule, and three parties. The p-function is assumed to be additive in the first example. Because of additivity, for reconstruction of the F function it is enough to show popular votes received by all parties under three-party coalition structures.

Party:	District 1	District 2	District 3
i	30	10	60
j	10	60	30
k	60	30	10

Table 2. Hypothetical E-function leading to split-merger stable three-party, single-member-district system.

In this environment, coalescing does not pay (although it does not hurt, either). Two coalescing parties will always get two seats, a number equal to the sum of the seats they could win separately.

In the example above there are no dramatic changes in voters' support when the most favorable party enters some coalition. In this case voters vote for their most preferred party

[7] Riker (1976), Cox (1987), and Palfrey (1989) founded their formal models of the mechanism behind the Duverger's Law on the 'wasted votes' or strategic voting assumption and presented them inside the spatial framework.

whether in a coalition or not. The stability of the three-party system is not due to some strange behavior of the voters, but rather due to important differences in the support among districts such as the deep M-district cleavages in Canada. These cleavages completely eliminate the possibility of any gains from coalescing.

One may also notice that in our example CS-payoff function could be substituted by a characteristic function. This is always possible when the F function itself is additive. Consequently, the game is inessential. Every coalition structure is not only split-merger, but also totally -stable. However, an example of an essential game with only three-party coalition structure stable is also easy to cook up. Let us assume the following p-function:

Party/coalition	Coalition structure	D1	D 2	D 3
i	{i},{j},{k}	30	30	40
j	{i},{j},{k}	40	30	30
k	{i},{j},{k}	30	40	30
j,k	{i},{j,k}	49	49	49
i,k	{j},{i,k}	49	49	49
i,j	{k},{i,j}	49	49	49

Table 3. Hypothetical &function leading to split-merger stable three-party, single-member-district system, with two-party system unstable.

Voters in this political environment "hate" coalitions, a lot of them switch from the coalescing parties to the remaining single one. Each coalition would get no seats at all, instead of two seats which could be assured by its members acting separately. The only split-merger-stable coalition structures are hence three-party and grand-coalition ones. The cleavages here are intra-district ones.

It is easier to construct examples of c-functions which produce two-party systems as split-merger-stable coalition structures but are unstable with three-party systems. The distributions of popular votes leading to such systems are also in the real world easier do find. This is why Duverger's Law is empirically important. In fact, in the real world they occur much more often. But the above examples show that there is no logical necessity for such constellation of popular votes. If the cleavages among districts are deep and stable over time and the strengths of a few parties are comparable and divided unequally among districts, plurality voting in single-member-districts can produce stable multi-party systems. Two simple theorems which bring some confirmation for this statement are included in the Appendix B.

The first theorem says that for a E-function representing additive popular votes, two-party systems are always split-merger stable under a plurality, single-member-district, voting rule. The second theorem says that two-party systems are the only split-merger stable systems if the additional assumption of "essentiality" of all the coalitions is added. One cannot expect these assumptions to be satisfied universally. However, one can speculate that the closer a p-function is to additivity, and the more evenly distributed popular votes of the coalitions in the districts, the greater the probability that the electoral system will be a two-party one. But "additivity" can be understood as the complete lack of "intra-district cleavages" where voters are indifferent about coalescing of their party. From the other side, "essentiality" is a certain measure of possible gains from coalescing, hence it can be understood as a situation, when "inter-district cleavages" are not deep enough to rule out all the possible gains from coalescing (like in the Table 2). Continuing the speculation, one can say consequently that the smaller the intra- and inter-district cleavages are, the more likely a stable two-party system will emerge under single-member-districts with plurality rule.

The above analysis resembles Riker's (1976) refmement of Duverger's Law. He stated that: "Plurality election rules bring about and maintain two-party competition except in countrics whcrc (1) third parties nationally are continually one of the two parties locally, and (2) one party among several is almost always the Condorcet winner in elections." The first clausc imposes certain inter-district restrictions, the second imposes certain restrictions on possible gains from coalescing by parties other than the distinguished Condorcet winner.[8]

Other reasons for explaining deviations from Duverger's Law can be also found by re-examining the assumptions underlying the current framework. Whenever there are no organizational or other gains from a strict merger of some parties as opposed to gains from

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[8] The importance of the clause of small "inter-district cleavages" was recognized even more explicitly by Rae (1971:96: "plurality formulae are always associated with two-party competition except where strong local minority parties exist." It seems that in general the importance of the "inter-district" cleavages enjoys wider appreciation by the scholars.

loose electoral coalescing in the political system, many parties can survive and form coalitions only before the forthcoming elections. The implicit assumption in the analysis is that a "merger" which can be identified with a "coalition" requires this type of additional benefits. The second implicit asumption in the model was that coalescing is possible at the national level only. If there are no legal or other obstacles to creating coalitions at lower levels (e.g., state), even a more complicated forms of the coalitional game would be required to capture the strategic complexity of the situation. In general, the incentives for merging under such arrangement would be greatly diminished. Last, but not least, whenever the intermediate level legislative bodies in large electoral systems (like state legislatures in India) create enough incentives for a local party to compete for votes, its objective can be quite different from "maximizing national level legislative seats" and chances for its survival may be not linked to it.

5. Coalition failures and parliament fragmentation. According to Riker (1982), "Duverger's Law" has to be distinguished from the "Duverger's Hypothesis" - a statement which says that Proportional Representation (PR) "favors" multi-party systems. Duverger's Hypothesis has some, but weaker, empirical support than Duverger's Law.

Multi-party PR electoral systems with thresholds and deep cleavages increases the strategic importance of making the right coalitional decisions. Failures in coalescing may result in obtaining less popular votes than necessary to cross the threshold. The wrong coalition may result in too few votes. The September 1993 Polish parliamentary (lower house) elections resulted in the- general failure of rightist parties to create large enough coalitions to win any votes in the general elections (see Table 4). Among the sources of the failure were (a) leaders' miscalculations in the face of a new formula; (b) the shift in electoral preferences (to left) after the deadline for announcing electoral coalitions; (c) last-minute creation of the rightist Walesa Bloc; (d) and the unexpected' electoral declaration of "Solidarity" which subtracted votes mostly from the rightist electorate.

Party/coalition	Pop. votes	Threshold	Seats
Center Alliance	**4.42%**	**5%**	**0**
Coalition "Fatherland"	6.37%	8%	0
Right Peasant's	2.37%	5%	0
" Solidarity "	**4.90%**	**5%**	**0**
Walesa Bloc	5.41%	5%	16
Libertarians	3.18%	5%	0
Radical anticommunists	2.70%	5%	0

Table 4. The 1993 electoral results of the rightist parties. Source: Monitor Polski.

The important source of the failure was the change of the electoral formula (which favors larger party) and introduction of the threshold (5 % for the parties, 8% for the electoral coalitions of the parties) which took place in the Spring 1993. This change did not give enough time for the fragmentized right to adjust. The previous formula (Hare with Droop quota and no thresholds) would have resulted in a completely different distribution of seats, with identical party behavior and identical popular votes (see Table 5). Under the 1991 voting rules, the right parties, even if divided, would have been quite successful and could have led to a coalition with a clear plurality in the parliament.

The electoral results in Table 5 are clustered by distances among the parties in the issue space, as evaluated by the author into "orientations". Only 17 of the biggest parties (out of 35) were included. Numbers in parentheses show the total number of parties included in the orientation. Column p contains proportions of popular votes obtained by different orientations (summed up), the 'actual seats' column contains actual distribution of seats, the 'simulated seats under 1991 rule' column contains a simulated distribution of seats under 1991 voting rule.

Orientation	p	actual seats	simulated seats under 1991 rule
Postcommunist (1)	20.4	171	98
Peasant (1)	15.4	132	75
Postsolidarity left (1)	7.3	41	44
Postsolidarity center (2)	14.6	74	72
Postsolidarity right (7)	29.4	16	123
Presolidarity right (1)	5.8	22	33
Other (4)	6.3	4	15

Table 5. Electoral results for 1993 parliamentary elections in Poland and simulated results under the 1991 voting rule. Source: *Monitor Polski* and simulation with the use of SEATS.

There were also minor coalition failures in the 1993 elections: between the two center postsolidarity parties and between the two extremist parties.

The coalition failures during the 1993 elections resulted in the distribution of legislative seats among smaller number of parties. The standard measure of parliamentary fragmentation, or the "number of parliamentary parties", was introduced by Laakso and Taagepera (1979) under the name of "effective number of parliamentary parties". This effective parties index, EP, assigns to every distribution of parliamentary seats a number according to formula (1):

$$EP(q_1,q_2,\ldots,q_n) = \frac{1}{\sum_1^n q_i^2} \quad (1)$$

where q_i is the proportion of parliamentary seats held by party i. Such a definition allows for capturing not only the total number of parties, but also their relative share. For an ideally uniform distribution, with every party holding $1/n$ proportion of seats, EP is equal to n. In the other extreme case, with one party holding a vast majority of the seats and a few others holding only tiny shares, EP assumes value close to 1. Table 6 shows the values of EP (real and simulated) for 1991 and 1993 Polish elections under 1991 and 1993 voting rules.

CS and p \ Voting rule	1991 voting rule	1993 voting rule
1991 CS and p	10.45	about 7.5
1993 CS and p	8.51	3.88

Table 6. Voting rule and parliamentary fragmentation. Sources: *Monitor Polski*, Gebethner (1992); and simulation with the use of SEATS.

Not surprisingly, substituting the 1991 Hare algorithm (which favors smaller parties) with the 1993 d'Hondt formula, thresholds and smaller districts, increases the effective number of parties in the parliament both for the 1991 and 1993 cases. However, under the special configuration of 1993 elections the change is especially large and a direct consequence of the coalitional failures of many parties, which left most of them outside of the parliament.

6. An operationalization of the 'Split-Merger Stability' hypothesis for the multiparty PR system. The 'Split-Merger Stability' hypothesis leads us to expect that under a stable voting rule the final coalition structure will be split-merger stable when the distribution of support is unchanged. However, with frequent and rapid endogenous changes in the voting rule, and with some variability in electoral support, there can be major departures from stability.

The 1993 Polish elections are a dramatic case of a non-stable coalition structure. The factors described in Section 5: change in the voting rule, shifts in the electoral support, and unexpected noncooperative moves of two players created a lot of uncertainty over the real values of electoral support. The failure was recognized by the rightist leaders a few weeks before the elections, but after the legal deadline for announcing electoral coalitions had passed. The only strategy left to them was to withdraw completely from the elections or to appeal to other rightist parties to withdraw. In fact, there were a couple of such appeals before the election date. However, after the legal deadline the electoral game change its structure to the noncooperative form. Any division of seats, 'which are the benefits from the collective action, was legally forbidden after the deadline. It means that a party withdrawing its candidates from the elections would get a zero payoff with certainty. But even with a small subjective probability of a success, the party would be better off taking this small chance. Therefore, the dominant strategy in this version of the collective action problem is to "not withdraw". All the rightist parties followed this strategy.'

The 'Split-Merger Stability' hypothesis suggests a major post-electoral adjustment toward greater split-merger stability. One can expect a couple of mergers between or among parties which would gain most from the coalescing. The empirical test of the 'Split-Merger Hypothesis' could be done in two parts:

1. The prediction based on a certain operationalization of the CS-game based on data for the Polish electorate;

2. The observation of the real political mergers/splits on the Polish political scene after elections.

The real political processes confirm the intuitive expectations. The rightist parties have formed two coalitions and are negotiating conditions of the mergers. The two center postsolidarity parties already formed the new party on April 24. Also the two small extremist parties began merger talks and are looking for partners.

The operationalization of the research question was set up before the elections and is based on the post-election survey to find out how the voters would have voted, if certain coalitions formed (see Appendix A). The complete reconstruction of the p-function is technically impossible for the all coalitions of the 17 largest parties, and will be done for pair-wise-merger coalitions only. This means that for every two-party coalition the total proportion of its supporters will be estimated as well as the preferences of the voters who would have decided to leave the coalition. The results of the survey will be summarized as a symmetric matrix 17x17 such that the number $F_{ij} = F_{ji}$ is defined as the estimated gain (loss) in the total number of the seats under the assumption that coalition of parties i and j formed, and all others didn't change their behavior. Therefore, the matrix F will allow to localize the regions of the greatest possible gains and to predict on this basis mergers of the parties in those regions.

The repetition of the research after subsequent elections in Poland will create an opportunity to reconstruct precisely a path of mergers between or among the parties and to

[9] The PD structure of this situation stems from the fact that the dominant strategy "not withdraw" is Pareto dominated by some mixed strategy, i.e., lottery over "not withdraw" and "withdraw".

compare it to the predictions stemming from the 'Split-Merger Stability' hypothesis. The 1989-1994 electoral path consists of three elections under different voting rules and the analysis of this path so far does not falsify the 'Split-Merger Stability' hypothesis.[10] However, the empirical test can be a very strong one in the case of the future elections, if the parties complete the first step of coalescing into clearly distinctive ideologically blocs, and the voting rule does not change.

7. A note on other possible applications of the model. The 'Split-Merger Stability' hypothesis is based on the assumption that the voting rule is stable and that there are no dramatic changes in the distribution of popular votes.

Holding only p constant for a given elections allows for a different type of research, namely, case studies of the so called mechanical effect of the voting rule. Such a research would be based on much more complex information about concrete elections rather than statistical analysis of a sample of countries or elections, which differ in electoral thresholds, algorithms, or average district magnitudes. The result would be the comparison of the possible eventual distributions of seats under different voting rules, like in Table 5. [11]

The scarcity of data and non-randomness of the sample makes a lot of inferences about the total impact of the electoral variables, like formulae, thresholds, and district magnitudes both statistically insignificant and not very reliable. From the other side, evaluating the impact of different voting rules on such variables like effective number of parties seems analytically intractable. A possible method of cutting this Gordian knot suggests simulation of possible distributions of popular votes (i.e., random generation of some values of the p function), calculation of distributions of seats and variables which characterize those distributions under different voting rules l, and treating the variables obtained in this "sample" with standard regression tools. The independent variables could be all the relevant voting rule components. The dependent variables could be such parameters of the electoral system as the effective and the actual numbers of parliamentary parties or their transformations.

The status of such a simulation can be similar to that of an experiment. When real

[10] See Appendix C.

[11] For a more detailed simulation based on 1991 Polish parliamentary elections see Appendix D.

data is scarce and some of the important independent variables kept outside the model, the estimators are inefficient and can be biased. Simulation can provide an important insight into the structure of relationships and their strength, as well as their robustness under different probability distributions.

Similar benefits can be obtained with the means of experiment. The subjects could get a complete description of the voting rule, each party's relative spatial positions and "poll" information about each party expected popular votes. They also could be allowed to negotiate up to some moment before the "elections". After the elections they would get more precise information about their unrealized gains. The "elections" would be then iterated a few times. Such experiments might throw some light on the dynamical aspect on the tatonnement process of achieving split-merger-stability.

Last but not least, the model can be developed in the spatial direction, The "global". spatial-coalition equilibrium would be a position in an issue space, under which parties neither have incentives to change the position, nor to coalesce further. The development of such a model would require a reliable theory that allowed the translation of relative spatial positions of coalescing parties into their seats-payoffs, an aspect missing in spatial models.

APPENDIX A

Questions in a post-poll survey (simplified translations from Polish).

1. Did you vote in the September elections? If YES:

2. What party or coalition did you choose?
The list of all parries is presented. Call the choice "A".

3. Please, imagine now that your party (coalition) A established a bigger coalition with some other party (coalition), and other coalitions did nothing. There are many possible coalitions here, we want you to express your opinion on all of them. Would you vote for A if its partner in such a coalition were:
A smaller lint of "big" parties (coalitions) is presented.

4. "A" is your first choice, the "real" one you voted for. What would be your second and third choice?
The same list as in 2. is presented.

APPENDIX B

All the below statements apply to the case of $\underline{l}$ defmed as a plurality rule in a single-member-district voting rule.

Lemma 1: If the p-function is additive, a merger of any number of coalitions cannot bring them a smaller number of seats than the sum of the seats they would get separately,

Proof: From additivity, the coalescing parties will get in every district not less than maximum of their popular votes in this district, and the other parties will get the same number of popular votes in this district. It means that the new coalition cannot loose a seat previously won in any district. Q.E.D.

Theorem 1: If the p-function is additive, the two-party system is split-merger stable.
Proof: Directly from Lemma 1. Q.E.D

Definition 1: A coalition $\underline{C}_i$ is essential if (a) there exists district $\underline{D}$ such that $\underline{C}_i$ can guarantee itself a majority of popular votes in this district; (b) there is no a proper subset $\underline{C}_j \subsetneq \underline{C}_i$ such that $\underline{C}_j$ can guarantee itself a majority of popular votes in district $\underline{D}$.

Theorem 2: If all the coalitions of two or more parties are essential and $\underline{p}$ is additive, the two-party systems are the only split-merger stable systems.

Proof: Assume that for certain m > 2, the m-party system is split-merger stable. Take a coalition C_i of any m-l parties. Let D be the district for which C_i is essential and let C_k denote the coalition which won the seat in D under the m-party system. From the essentiality assumption, C_k obtained less than 50% of popular votes in D. Two cases are possible:

a) Coalition C_k is among m-l coalitions from C_i. In this case take the remaining parties from C_i and let them coalesce with the m-th party. In all the districts except D they will get (from Lemma 1) at least as many seats as before coalescing and in district D they will get (lOO%-per cent of votes obtained by C_j) popular votes, hence majority. It means that this coalition can win an extra seat by a merger.

b) Coalition C_k is the m-th party. In this case the coalition C_i can get an extra seat in D.

In both cases m-party system cannot be split-merger stable for m > 2. Q.E.D.

From Theorem 2 we can easily obtain the following conclusion regarding dynamic properties of the CS-game:

<u>Corollary</u>: Take any m-party system. If all the coalitions of two or more parties are essential and $\underline{p}$ is additive, there exists a "path" of beneficial mergers leading to some split-merger stable two-party system.

APPENDIX C

POLISH 1989-1994 ELECTORAL PATH:

1989. **Voting rule:** two-seats-districts, two-nontransferable-votes, seats for two highest proportions.
Parties: ***Solidarity*** versus ***PUWP*** (communists).
Results: ***Solidarity*** won all lower house seats and 99 (out of 100) upper house seats by the average margin 3 : 1.

After the elections: There were many splits inside ***Solidarity*** which created a political "state of nature". A new voting rule, favorable for smaller parties, was established. The project of plurality, single-member-district rule for most, seats by the largest party was defeated by the coalition of all the other parties (postsolidarity and postcommunist).

1991. **Voting rule:** big districts, Hare formula, no thresholds.
Parties: About 200 registered, more than 50 in elections.
Result..: More than 20 parties won seats in the parliament, **EP=10.45.**

After the elections: a few splits, a few mergers. Spring 1993: the major parties voted together for a new voting rule which was much more favorable for larger parties.

1993. **Voting rule:** 5 % threshold for a single party, 8 % for a coalition, 7% for a nationwide list; smaller districts, **d'Hondt** formula.
Parties: About **35** parties in the elections, 15 registered nationwide.
Results: 6 parties in the parliament (**+4** seats minority), **EP =3.88.**

After the elections: intensive merger talks **occured.** Two rightist coalitions created, a new center postsolidarity party (from two previous parties) established on April 25, 1994. Talks between the two extremist parties began.

APPENDIX D

SIMULATED MECHANICAL EFFECT FOR 1991 ELECTIONS

Index EP^P is the effective number of parliamentary parties; EP^V is the effective **number of popular votes (like EP^P, but based on the distribution of popular votes). The ratio** EP^V/EP^P is a measure of a voting rule's ability to convert popular votes into parliamentary **seats.**

Method	Threshold	EP^P	EP^V/EP^P
Hare (used)	no	10.45	1.11
PR (one list)	**no**	**11.61**	**1**
Hare	district: 3% 5% 10% nationwide: 5 % 10%	**9.88** **8.51** 7.07 8.63 2	1.18 1.37 1.64 1.35 5.8
Saint-Lag&	no district: 5% nationwide: 5 %	8.87 8.43 8.25	1.31 1.38 1.41
d'Hondt	no district: 5 % 10% nationwide: 5 %	8.69 8.40 7.12 8.25	1.34 1.38 1.63 1.41
"veto"*	no.	10.48	1.11
Mixed majority-PR**	no	6.64	1.75
winner-take-all	no	**5.71**	2.03
Hare with more subtle district division (52)	no district: 5%	9.72 8.53	1.19 1.36

Table 7. Actual and simulated electoral measures for 1991 parliamentary elections in Poland. Source: Gebethner (1993), simulation with the use of SEATS.

* The electoral system accepted by **Parliament** on May 11, 1991, and subsequently vetoed by the President.

** A project to assign 391 seats in single-member districts by plurality and 69 seats from a national list -- proposed by the formerly largest party and rejected by all others.

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